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Technical Change, Income Distribution, and Profitability: Marx's Law Revisited *

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Abstract: Under the assumption of constant real wage, Okishio's theorem shows that profit rates do not fall after any viable technical change. Research has indicated that if real wages rise after the introduction of technical change and then profit rates fall, then such fall in profit rates belongs to the realm of profit squeeze theory, which leads to the claim of the impossibility of a consistent theory of declining profit rate based on Marx's insight. The present study proposes a two-channel framework to distinguish the mechanism of rising organic composition of capital from that of profit squeeze, and show that any viable capital-using and labor-saving technical change would lower the profit rate if the wage/profit ratio is unaffected in a multi-sector setting.

Keywords: technical change; income distribution; falling rate of profit; Okishio's theorem

1 Introduction

The history of capitalism has seen dramatic technological revolutions and the permanent improvement of living standards, on the one hand, and constant conflicts over the distribution of income and cyclical recessions and crises on the other. As shown in the history of mechanization and ongoing spread of automation, new techniques reduce the labor input required in production with the help of machines, robots, and even artificial intelligence. These capital-using and labor-saving (CU-LS) technical innovations enable the growth of real wages and rising living

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standards. However, they also tend to increase the capital/labor ratio and drive down profitability, which has significant impacts on investment and stability and contributes to cyclical recessions and crises. This is one of the main insights on the dynamics of capitalism provided by the classical political economy (Kurz, 2010), as summarized by Marx (1993) in his "law of the tendency of the rate of profit to fall" (Marx's law hereafter)—technical change in capitalism tends to bring about a falling profit rate due to the rising organic composition of capital (OCC). However, this "law" is conventionally dismissed as either a failed empirical prediction or an inconsistent theoretical argument.

Acemoglu and Robinson (2015) refuted Marx's law using empirical evidence and asserted that such a general law fails because it ignores the role of institutions and politics. It is true that the historical trajectory of the profit rate is shaped by other factors, as acknowledged by Marx in the discussion of the "counteracting influences". However, the point is whether such an economic force exists and drives down profitability, no matter how it may be influenced by other counterbalancing forces.

The real challenge for Marx's law came from Okishio(1961)'s theorem, which cast doubts on the internal consistency of Marx's argument under conditions of competitive capitalism (Roemer, 1977). Okishio (1961) showed that the profit rate rises after any cost-reducing technical change if the real wage is unaffected. Arguably, Okishio's theorem does not necessarily refute Marx's law, since it is possible to have a falling profit rate if the real wage rises sufficiently (e. g., Roemer, 1978; Laibman, 1982).

However, van Parijs (1980) claimed that relaxing the assumption of fixed real wage does not help in the construction of a theory of falling profit rate due to rising OCC. The rate of profit may fall due to the rising real wage, as "the erosion of profits as a result of successful class struggle waged by labor against capital" (Boddy and Crotty, 1975, p. 1). Such a mechanism has been addressed by the "profit squeeze" argument and has nothing to do with the rising OCC. Here is the dilemma: if real wage is unaffected after the introduction of technical change, then the profit rate rises according to Okishio's theorem; if real wage rises after the introduction of technical change and the profit rate falls, then the profit squeeze argument applies. Therefore, van Parijs concluded that a theory of falling profit rate due to rising OCC is "not even a possibility" in his assessment of Marx's law (van Parijs, 1980, p. 1).

Is it possible to isolate the two mechanisms of falling profit rate? Is there such an underlying economic force driving down profitability due to rising OCC based on Marx's insight? The present study seeks to address these questions. The answers to them may well be positive. If no change occurs in the power relations such that the wage/profit ratio is constant after the introduction of technical change, then profits are not squeezed by the rising real wage, given that the wage/profit ratio is unaffected. Thus, any change in the profit rate cannot be explained by the profit squeeze argument. Therefore, the assumption of a constant wage/profit ratio allows the separation of the mechanism of rising OCC from that of profit squeeze.

Further, under the assumption of a constant wage/profit ratio, the rate of profit falls after viable CU-LS technical change. This clear-cut result supports Marx's law as an underlying economic force in capitalism. The viable CU-LS technical innovation itself (in the absence of power change) would drive down profitability, even though it is profitable under current prices and would be adopted by rational capitalists.

The remainder of this paper is organized as follows. Section 2 outlines the research strategies and summarizes the main results for non-math-oriented readers, whereas a formal and rigorous presentation is provided in Section 3. Section 4 briefly discusses the related literature and highlights the differences between the results obtained in the present study and those in existing literature. Some directions for further study are suggested in the last section, and the proofs of the formal results are presented in the Appendix.

2 Effect of Technical Change on Profitability: Verbal Exposition

Following the convention of existing analyses, the author considers the economy with Leontief technology and focused on the long-run outcome defined by a uniform rate of profit—the situation after the transformation of labor value into price of production. In exploring the consequences of technical change on profitability, the standard strategy is to conduct a comparative static analysis by comparing the long-run outcome after the technical change with the initial one. However, as suggested in the Marxist political economy and confirmed by recent studies on Sraffian indeterminacy (e.g., Mandler, 1999; Yoshihara and Kwak, 2019), the long-run outcomes cannot be determined simply by the technology and competitive market; additional conditions should be introduced.

2.1 Three Benchmarks

Three different assumptions have been proposed in the following three benchmarks below:

(i) Okishio's scenario assuming constant real wages;
(ii) Sraffian scenario assuming constant rate of profit; and
(iii) Marxian scenario assuming constant wage/profit ratios.

The first one is adopted in Okishio's seminal paper in which he provides the first modern formalization of Marx's insight using matrix algebra (Okishio, 1961). The second is conventionally made in the literature on the choice of technique in the Sraffian tradition (e.g., Sraffa, 1960; Pasinetti, 1977). The last one is inspired by the literature of decomposition of profit rate① and Marx's own argument on falling profit rate due to rising OCC, in which Marx assumed that the distributional situation is unaffected after the introduction of technical change or that "the rate of surplus-value, or the level of exploitation of labour by capital, remains the same" (Marx, 1993, p. 318)②. Instead of assuming a constant rate of exploitation and working with

① See the classical paper by Weisskopf (1979), in which he argues that Okishio Theorem "does not discredit on *a priori* grounds the ROC [rising organic composition of capital] variant of Marxian crisis theory" because "only a constant share of wages in national income" is assumed in his framework (343f). Since that early work, a vast literature has employed his methodology to analyze the behavior of profit rate (e.g., Cámara Izquierdo, 2013), which will be briefly discussed in Section 4.

② What does Marx hold constant before and after technical change? Okishio (1961)'s answer is real wage rate and therefore he seems to take his main result, the so-called Okishio Theorem, as a negation to Marx's law. However, Marx summarizes his law as "the same rate of surplus-value, therefore, and an unchanged level of exploitation of labour, is expressed in a falling rate of profit" (Marx, 1993, p. 317), which indicates that it is the rate of surplus-value that Marx hold constant.

the value system, as in Laibman (1982) and Bidard (2004, p. 73-76), I fix the sectoral wage/profit ratio in terms of prices in the Marxian scenario.

All of these benchmarks could be the outcome of a competitive market. Typically, only Okishio's scenario is taken as the default assumption of a competitive market (e.g., Roemer, 1977), despite the absence of convincing reason to do so. Real wage cannot be determined by the competitive market alone because of the factor/price indeterminacy and the dependence of wage rate change on some non-market factors, such as the power relations between the two classes of labor and capital. Even if real wage is assumed to be regulated by a competitive labor market, real wages may not necessarily remain unaffected after the introduction of technical change (Yoshihara and Veneziani, 2019). Therefore, the assumption of a constant real wage in Okishio's scenario is one specific case, and the other two scenarios could also be supported by a competitive market under different conditions of those non-market factors.

The effects of viable CU-LS technical changes on profitability in the three scenarios are summarized in Table 1①. As shown in the second column, the OCC in the sector where technical change occurs increases in all cases. However, the profit rate may rise, remain constant, or fall. In Okishio's scenario with real wage unaffected, the viable CU-LS technical innovation changes the distributional situation against workers—wage/profit ratio falls in the sector with technical change, and the rate of exploitation rises overall, leading to an increase in the rate of profit. In the Sraffian scenario, where the profit rate remains constant, real wages rise, but the wage/profit ratio still falls in the sector with technical change. In contrast, the profit rate falls in the Marxian benchmark where real wages rise sufficiently to maintain the wage/profit ratio after the introduction of technical change.

Table 1 Effects of viable capital-using labor-saving technical change in sector *j*.

	Composition of capital (q_j)	Wage/profit ratio (γ_j)	Profit rate (π)
Okishio's scenario	↑	↓	↑
Sraffian scenario	↑	↓	fixed
Marxian scenario	↑	fixed	↓

2.2 Two-Channel Framework

As mentioned in the Introduction, the main results in the three scenarios suggest an argument against the claim that it is impossible to build a consistent theory of falling profit rate due to rising OCC. Below I set up a two-channel framework and demonstrate how the three scenarios fit together to support Marx's insight within this framework.

As shown in Figure 1, technical change influences the rate of profit through two channels: (i) the change in the composition of capital and (ii) the change in power relations. On the one hand, the CU-LS technical change itself (in the absence of change in power relations) would raise the OCC and drive down the rate of profit.

① The precise statements and proofs of these results are presented in Section 3 and the Appendix, respectively.

On the other hand, the technical change may weaken the bargaining power of the working class (e. g. , by replacing skilled labor with machines or monitoring workers with the help of AI) and hence raise the degree of exploitation and rate of profit. The first corresponds to "the law as such" and the second is one of the "countering influences" in Marx's law (Marx, 1993, p. Chapter 13-14).

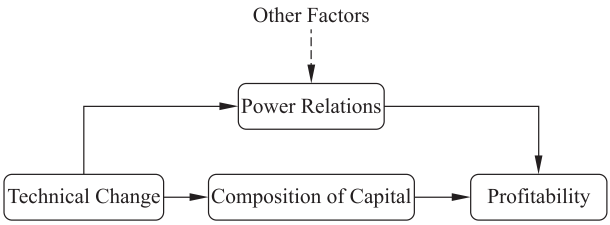

Figure 1　Two channels through which technical change impacts profitability

Other factors may also change the social power structure and thus impact profitability. The idea that the change in working class power would shape the movement of the rate of profit is suggested in volume one of *Capital* (Marx, 1992) and is formalized as the profit squeeze argument, either the short-run version proposed by Boddy and Crotty (1975) or the long-run one by Glyn and Sutcliffe (1972).

This framework shows how profit rate and OCC rise together after viable CU-LS technical change in Okishio's scenario. As indicated by the falling wage/profit ratio and the raising rate of exploitation, technical innovation does not just change the OCC but also the distributional situation. Therefore, the total effect of technical change on profit rate is the composition of the effects through both channels. Okishio Theorem simply says that after viable technical change "capitalistic classes can raise the rate of profit, if laborers fail to get an increase of wages" (Okishio, 1961, p. 96). In other words, if all the benefits from the viable technical progress are taken by the capitalists, then profit rate would rise even when such a technical change is CU-LS and raises the OCC. In this perspective, Okishio Theorem shows that with unaffected real wages the effect of the second channel counteracts and dominates the effect of the underlying force of driving down profitability due to rising OCC, which does not refute Marx's law.

van Parijs (1980) argued that fixed real wages must be assumed when constructing a theory of falling profit rate due to rising OCC. Otherwise, even if the profit rate falls, the scenario is not a consequence of the rising OCC but the result of the increasing working class power, which falls under the realm of the profit squeeze argument (van Parijs, 1980, p. 4). However, this is not always true: With technical progress, the surplus is changing such that an increase in the real wage rate is not sufficient to capture the change in distribution. For example, in the Sraffian scenario, workers succeed in increasing real wages but fail to maintain the sectoral wage/profit ratio after the introduction of technical change. That is, even though the real wage rates rise, technical innovation still changes the distribution situation against workers as in Okishio's scenario. Therefore, in the context of technical change, rising real wage does not indicate an increase in working class power.

In effect, to examine whether Marx's insight holds, we should block the channel of power relations so that any change in profitability cannot be explained by the profit squeeze argument. In the Marxian benchmark, by considering the technical change as power neutral such that the wage/profit ratios are unaffected, we find that profit rate falls as the composition of capital increases. In short, technical change *itself* (that is, in the absence of power relation changes)① brings about a falling rate of profit as a result of a rising OCC.

3 Model and Formal Analysis

3.1 Long-run Outcome and Technical Change

Consider an economy with linear technology (A, l), where A is an $n \times n$ input/output matrix whose element a_{ij} represents the amount of goods i needed in the production of one unit of goods j, l is a row vector, and l_j is the direct labor required in the production of one unit of goods j. Suppose that (A, l) satisfies the standard assumptions—the nonnegative matrix A is productive and indecomposable, and labor is indispensable②. Let p be the price row vector and w the wage rate row vector, where w_j is the wage rate in industry j. Let L denote the diagonal matrix of direct labor inputs, i.e., $L = \text{diag}\{l_1, \cdots, l_n\}$.

We focus on the long-run outcome with a uniform rate of profit π such that

$$p = (1 + \pi)(pA + wL) \tag{1}$$

and two associated properties in the long-run outcome. The first property is the wage/profit ratio γ defined by

$$\gamma_j = \frac{w_j l_j}{\pi(pA^j + w_j l_j)} \text{ for all } j$$

The second one is the OCC defined by

$$q_j = \frac{pA^j}{pA^j + w_j l_j} \text{ for all } j$$

which is the counterpart of $q = c/(c+v)$ following Sweezy (1942), where c and v are constant and variable capitals in terms of value, respectively③.

Formally, the long-run outcome is denoted by (p, π, γ, q) given technique (A, l) and wage w. It captures the classical idea that with free competition the actions of profit-seeking producers would bring about a tendency toward a uniform rate of profit. However, the notion of long-run outcome in this paper differs slightly from the

① Roemer (1979, p. 385) interpreted Okishio's theorem as the negation of the claim that "technical change *itself* (that is, in the absence of real wage change) can bring about a falling rate of profit in consequence of a rising OCC". In contrast, I take "technical change itself" as the absence of power relation change.

② Mathematically, this study assumes that (i) A is productive: there exists a nonnegative vector $x>0$ such that $x\text{-}Ax>0$; (ii) A is indecomposable: for all i, j, there exists t such that $(A)^t_{ij} \gg 0$; and (iii) labor is indispensable $l \gg 0$. For two vectors u, v with the same dimension, $u \geqslant v$ means $u_i \geqslant v_i$ for all i, $u \gg v$ means $u_i > v_i$ for all i, and $u > v$ means that $u \geqslant v$ but $u \neq v$.

③ There are many alternative definitions of the composition of capital and I would refer the readers to van Parijs (1980, p. 13n5) for a survey. One might consider $c/(v+s)$ as a better formula since it is invariant to distributional change, while others believe it is misleading to call the ratio of dead labor to living labor as the composition of capital because the denominator $(v+s)$ is not capital advanced (e.g., Gao, 1991, p. 53). Furthermore, some authors argue that in a dynamic context it is better to define the organic composition of capital using the prices/values at the initial period (e.g., Saad-Filho, 1993), and the definition we adopt here should be termed as the value composition of capital. However, the various formulas and terminologies will not change our analysis and the main results in this paper.

conventional long-period equilibrium or long-period position, which is characterized not just by a uniform rate of profit but also the "uniform rates of remuneration for each particular kind of 'primary' input in the production processes, such as different kinds of labor and natural resources" (Kurz and Salvadori, 1995, p. 1). Therefore, the long-run outcome can be taken as a generalization of long-period position with persistent wage differentials among sectors even with homogeneous labor (Botwinick, 1993). It allows us to consider the case with constant sectoral profit/wage ratios, and hence separate the mechanism of rising OCC from that of profit-squeeze.

Consider a technical change in sector j from (A^j, l_j) to (A^{*j}, l_j^*), with the new technology denoted by (A^*, l^*). Following the existing literature, the following can be defined:

(i) The technical change is viable (or cost reducing) if $pA^{*j}+w_j l_j^* < pA^j + w_j l_j$.

(ii) It is CU-LS if $A^{*j} > A^j, l_j^* < l_j$.

(iii) It is progressive if $v^* < v$, where $v = l(I-A)^{-1}$ and $v^* = l^*(I-A^*)^{-1}$ are the labor values. Roemer (1977) showed that if a technical change is both viable and CU-LS, then it must be progressive.

The concept of viable technical change, introduced by Okishio (1961), has been widely adopted as a criterion of technical choice. The definition of CU-LS technical change following Morishima (1973, p. 137) is believed to capture the technological innovations that replace labor with machines and enhance labor productivity, or in Marx's terminology, technical changes that occur with a rising "technical composition of capital" (Marx, 1993, p. 244). This type of technical change is relevant not only to the history of capitalism but also to the worldwide trend of automation. Therefore, the present study focuses on viable CU-LS technical changes.

The new long-run outcome is denoted by $(p^*, \pi^*, \gamma^*, q^*)$ given the new technique (A^*, l^*) and wage w^*.

3.2 Comparative Analysis of Three Scenarios

To explore the consequences of technical change, this study compares the two long-run outcomes, (p, π, γ, q) and $(p^*, \pi^*, \gamma^*, q^*)$, before and after the introduction of innovation①.

In Okishio's scenario, real wage is given by matrix B whose column b^j denotes the real wage bundle in sector j, which is assumed to be unaffected after the introduction of technical change. In other words, the wage rates are given by $w_j = pb^j$ and $w_j^* = p^* b^j$ for all j, or equivalently, $w = pB$ and $w^* = p^* B$. The main results of Okishio's scenario are summarized in the following proposition②.

Proposition 1. Let (p, π, γ, q) be the long-run outcome of the economy with (A, l) and $w = pB$. Consider a technical change in sector j in Okishio's scenario and let $(p^*, \pi^*, \gamma^*, q^*)$ be the new long-run outcome with (A^*, l^*) and $w^* = p^* B$. Then,

(i) The profit rate rises $(\pi^* > \pi)$ if and only if the technical change is viable.

① The discussion on the existence of long-run outcomes is presented in the Appendix.

② See the Appendix for the proofs of the propositions in this section.

(ii) If the technical change is viable, then the price of goods j decreases the most. That is, $\frac{p_j^*}{p_j} < \frac{p_i^*}{p_i}$ for all $i \neq j$.

(iii) If the technical change is viable and CU-LS, then the wage/profit ratio falls and OCC rises in sector j, i. e., $\gamma_j^* < \gamma_j$ and $q_j^* > q_j$.

(iv) If the technical change is viable and CU-LS, then the value of labor power falls, $v^* B \ll vB$.

Part (i) generalizes the well-known Okishio's theorem and its converse (Dietzenbacher, 1989) into the case where real wage rates vary among sectors. Part (ii) shows the effect of technical change on relative prices and plays a key role in establishing part (iii) because both the wage/profit ratio and OCC are measured in terms of prices. Parts (iii) and (iv) show that workers are relatively worse off after a viable CU-LS technical change with constant real wages. If we define the rate of exploitation in sector j by $e^j = (1 - vb^j)/vb^j$, then part (iv) implies that the rates of exploitation rise in all sectors.

In the Sraffian scenario, the nominal wage rates are allowed to adjust to maintain the rate of profit after the introduction of technical change. Mathematically, the problem is to find a new wage rate vector $w^* \in \mathbb{R}_+^n$ such that $\pi^* = \pi$ at the new long-run outcome, and there might be multiple solutions. In the following proposition, we fix the structure of the wage rate vector, i. e., $w^* = \alpha w$, and only its magnitude is allowed to adjust after technical change[①].

Proposition 2. Let (p, π, γ, q) be the long-run outcome of the economy with (A, l) and w. Consider a technical change in sector j in the Sraffian scenario and let $(p^*, \pi^* = \pi, \gamma^*, q^*)$ be the new long-run outcome with (A^*, l^*) and $w^* = \alpha w$. Therefore,

(i) If the technical change is viable, then prices in terms of wage falls, $\frac{p^*}{\alpha} \ll p$.

(ii) If the technical change is viable and CU-LS, then the wage/profit ratio falls and OCC rises in sector j: $\gamma_j^* < \gamma_j$ and $q_j^* > q_j$.

(iii) If the technical change in sector j is viable and CU-LS, then the wage/profit ratio rises in other sectors: $\gamma_i^* > \gamma_i$ for all $i \neq j$.

Part (i) shows that prices in terms of wages fall after the introduction of viable technical change. This implies that the real wage rate rises regardless of which bundle of goods is chosen as the standard, as reported in the Sraffian literature (e. g., Sraffa, 1960; Pasinetti, 1977, pp. 158-159). However, the fixed profit rate and rising real wage rate do not indicate a relative improvement in workers.

Parts (ii) and (iii) show that with a fixed profit rate, the wage/profit ratio in the sector with technical change would fall, whereas that in other sectors would rise. An implication is the ambiguity of the overall effect on the distribution. The aggregated wage/profit ratio (or wage share) in the economy as a whole—the weighted average of the ratios in different sectors—could either increase or decrease after the introduction of technical change, which depends on the structure of outputs. For example, if a technical change occurs in a sector that hires the majority of the labor force, then a fall in the sectoral wage/profit ratio could lead to a decline in the

① Note that wage rate is usually assumed to be uniform in the standard Sraffian literature, which can be taken as a special case of our setting.

overall wage share. Without an explicit assumption on the structure of output, the overall distributional effect of technical change cannot be presumed.

To avoid this complexity and indeterminacy brought about by the structure of outputs, I fix the *sectoral* wage/profit ratios rather than the overall wage share as a benchmark in the Marxian scenario. Assuming that technical change does not change the relative power of capitalists and workers, the bargaining between these two classes can find a new long-run outcome with constant wage/profit ratios after the introduction of technical change. That is, the wage rates can be adjusted to maintain the wage/profit ratios, or $\gamma_j^* = \gamma_j$ for all j.

Proposition 3. Let (p, π, γ, q) be the long-run outcome of the economy with (A, l) and w. Consider a viable CU-LS technical change in sector j and let $(p^*, \pi^*, \gamma^* = \gamma, q^*)$ be the new long-run outcome. Then the profit rate decreases with increasing OCC, $\pi^* < \pi$ and $q_j^* > q_j$.

Proposition 3 shows that the rate of profit decreases with rising OCC after the introduction of viable CU-LS technical change if sectoral wage/profit ratios are unaffected. The intuition of this result is exactly Marx's insight on falling profit rate as a result of rising OCC, although his argument accounted for value at an aggregated level. Sweezy (1942, p. 68) formulated Marx's argument as follows: If the rate of exploitation $e = s/v$ is fixed, and OCC $q = c/(c+v)$ increases, then the general rate of profit r falls:

$$r = \frac{s}{c + v} = \frac{s}{v}\frac{v}{(c + v)} = e(1 - q)$$

Proposition 3 translates this idea into a world with prices. The CU-LS technical change could reduce the total wage relative to total costs. Consequently, per the constant wage/profit ratio, profit is also reduced relative to the total cost, and thus, by definition, the profit rate falls.

4 Related Literature

Proposition 3 is a generalization of Roemer (1978) in a multi-sector model. Roemer obtained similar results in a simple two-goods two-sector model with adjustment in real wages to maintain a constant wage share. Skillman (1997) provided a micro-foundation of constant wage share by assuming that wages are determined by sequential bargaining within a stationary matching process in a one-good model.

Julius (2009) also observed a falling profit rate after technical change when the bargaining power is unchanged. Julius showed only the existence of such technical change, which differs from the present results with respect to all viable CU-LS technical change. Instead of fixing the wage/profit ratios, Laibman (1982) and Liang (2021) assumed a constant rate of exploitation in reference to some consumption bundles, but failed to obtain determinate results unless the technical change occurs in the consumption goods sector. A falling profit rate could also be observed by modifying other assumptions in Okishio's setting. For example, Shaikh (1980) developed models in which capitalists focus on profit margins instead of profit rates, whereas Skott (1992) and Michl (1994) formulated models with imperfect competition. The current results show that Marx's law holds as an underlying mechanism that does not depend on any imperfection of goods market or *ad hoc* assumption of behavior of the capitalist.

Proposition 2 differs from the model of Franke (1999) with fixed capital stock, in which wage share falls if the rate of profit remains fixed. First, Franke considered the aggregated wage share in a balanced growth path instead of the sectoral wage/profit ratio. Second, Franke's result may hold only for *ad hoc* technical change with increasing fixed capital stock and uniformly decreasing labor input in all sectors.

One of the main purposes of this paper is to isolate different mechanisms of falling profit rate, which is related to the strand of empirical literature on the decomposition of profit rate pioneered by Weisskopf (1979). In his empirical framework, profit rate can be decomposed as the product of the profit share, the capacity utilization rate and the capacity/capital ratio as an identity in aggregate terms. It does not isolate different mechanism of declining profit rate theoretically and it does not aim to do so[①]. As Weisskopf pointed out, technical change may not result in a rise in the organic composition of capital because the prices would also change, which "clearly cannot be resolved *a priori*" in his framework (Weisskopf, 1979, p. 344). Indeed, the price effects can only be explicitly examined in a multi-sectoral model. As shown in this paper, the sectoral organic composition of capital rises after CU-LS viable technical change when wage/profit ratios are assumed to be constant. Therefore, the results in this paper can be taken as a support for Weisskopf's methodology to some extent. However, we are only at the halfway mark since Proposition 3 only deals with the price effects without considering the changes in output structure brought by technical innovation.

5 Concluding Remarks

This research examines the effects of viable CU-LS technical changes on income distribution and profitability by considering three scenarios with fixed real wages, constant profit rate, or unchanged distribution. The CU-LS technological innovations adopted by capitalists do not increase the rate of profit unless they are power biased against the working class such that wage/profit ratios cannot be maintained. Thus, a technical change in the absence of power relation changes would bring about a falling rate of profit due to rising OCC.

There are several possible extensions to this study. For example, instead of focusing on an economy with single production and pure circulating capital, future research may consider an economy with joint production. It is possible to have a falling profit rate with fixed real wages and joint production (Salvadori, 1981; Woods, 1984). Meanwhile, Okishio's theorem can be extended to the case of pure fixed capital or joint production within von Neumann's framework (Roemer, 1979; Nakatani, 1980; Woods, 1985). Indeed, the crucial assumption in generalizing Okishio's theorem is the existence of a positive standard commodity (Bidard, 1988). With this assumption, whether the argument on a falling profit rate due to rising OCC could be generalized remains an open question.

Another natural extension of this work would be to incorporate the change in output structure brought by technical innovation and explore (i) the effect of technical change on the overall wage share instead of the sectoral wage/profit ratio in

① Instead, the purpose is to identify the *initial source* of decline in the rate of profit and then determine the extent to which each mechanism explains the decline plausibly (Weisskopf, 1979).

the Okishio's and Sraffian scenarios; and (ii) the effect of technical change on the rate of profit when holding the wage share constant.

Appendix

Notations. For two vectors u, v with the same dimension, $u \geqslant v$ means $u_i \geqslant v_i$ for all i, $u \gg v$ means $u_i > v_i$ for all i, and $u > v$ means that $u \geqslant v$ but $u \neq v$. Denote the all-ones vector by $\mathbf{1} = (1, \cdots, 1)$.

Matrix expression. Let $\Gamma = \text{diag}\{\gamma_1, \cdots, \gamma_n\}$ be the diagonal matrix of the wage/profit ratios. By the definition of wage/profit ratios, $wL = \pi(pA + wL)\Gamma$. According to Equation (1), $pA + wL = p/(1+\pi)$. Therefore,

$$wL = \frac{\pi}{1+\pi} p\Gamma \tag{2}$$

and hence, $p = (1+\pi)(pA + wL)$ can be rewritten as

$$p = p[(1+\pi)A + \pi\Gamma] \tag{3}$$

Lemma A. Let the nonnegative matrix $M > 0$ be indecomposable, and r be its Frobenius root. If for a row vector $p > 0$, $pM < p$ (resp. $>$), then $r < 1$ (resp. $>$).

Proof. Let $p_0 \gg 0$ be a positive characteristic (row) vector of M' associated with r: $p_0 M' = r p_0$. $pM < p$ and $p_0 > 0$, therefore, $pMp_0' < pp_0'$, i. e., $rpp_0' < pp_0'$, and since $pp_0' > 0$, $r < 1$.

Existence of long-run outcomes. Given real wages matrix B, if $vB < \mathbf{1}$, then there exists $p > 0$ and $\pi > 0$ such that $p = (1+\pi)p(A + BL)$. In other words, if the rate of exploitation $e_j = \frac{1}{vb^j} - 1$ is nonnegative for all j and is positive in at least one sector, then there exists a long-run outcome associated with a positive profit rate.

Proof. Let $M = A + BL$. We show that $vB < \mathbf{1}$ implies $<v$. $vM = vA + vBL < vA + \mathbf{1}L = vA + l = v$. Then the Frobenius root $r < 1$ by *Lemma A*, which ensures the existence of the long-run outcome.

Proof of Proposition 1

Let $M = A + BL$ and $M^* = A^* + BL^*$. Then M^* differs from M only in the jth column, i. e., $M^i = M^{*i}$ for all $i \neq j$. Let $r = 1/(1+\pi)$ and $r^* = 1/(1+\pi^*)$ be the Frobenius root, respectively: $pM = rp$ and $p^* M^* = r^* p^*$, i. e., $\frac{p \cdot M^k}{p_k} = r$ and $\frac{p^* \cdot M^{*k}}{p_k^*} = r^*$ for all k.

For part (i), by the Collatz-Wielandt formula [see e. g., Theorem B in Roemer (1977, p. 422)],

$$\min_k \frac{p \cdot M^{*k}}{p_k} < r^* < \max_k \frac{p \cdot M^{*k}}{p_k}$$

First, we show that if the technical change is viable, then $r^* < r$. Since the new technology is viable, $p \cdot M^{*j} < p \cdot M^j$ and then $\frac{p \cdot M^{*j}}{p_j} < \frac{p \cdot M^j}{p_j} = r$. Since $\frac{p \cdot M^{*k}}{p_k} =$

$\frac{p\cdot M^k}{p_k}=r$ for all $k\neq j$, it follows that $r^*<\max_k\frac{p\cdot M^{*k}}{p_k}=\frac{p\cdot M^j}{p_j}=r$ and hence $\pi^*>\pi$.

For the converse, we show that if the technical change is not viable, i. e., $p\cdot M^{*j}\geqslant p\cdot M^j$, then $\pi^*\leqslant\pi$. If $p\cdot M^{*j}=p\cdot M^j$, then $pM^*=pM=rp$. Therefore, $r^*=r$ and hence $\pi^*=\pi$. If $p\cdot M^{*j}>p\cdot M^j$, then $\frac{p\cdot M^{*j}}{p_j}>\frac{p\cdot M^j}{p_j}=r$. Since $\frac{p\cdot M^{*k}}{p_k}=\frac{p\cdot M^k}{p_k}=r$ for all $k\neq j$, it follows that $r=\min_k\frac{p\cdot M^{*k}}{p_k}<r^*$ and hence $\pi^*<\pi$[1].

For part (ⅱ), we prove it by contradiction. Suppose on the contrary that the price of goods j does not decrease the most, i. e., there exists some $k\neq j$ such that

$$\frac{p_k^*}{p_k}\leqslant\frac{p_i^*}{p_i}\text{ for all } i$$

That is, $\frac{p_i}{p_k}\leqslant\frac{p_i^*}{p_k^*}$ for all i. Let $\bar{p}=p/p_k$ and $\bar{p}^*=p^*/p_k^*$, then $\bar{p}^*\geqslant\bar{p}$. By the definition of long-run outcomes, we have $(1+\pi)\bar{p}M=\bar{p}$ and $(1+\pi^*)\bar{p}^*M^*=\bar{p}^*$. Note that $M^{*k}=M^k$ since $k\neq j$. It follows that $\bar{p}^*M^{*k}=\bar{p}^*M^k\geqslant\bar{p}M^k$ since $\bar{p}^*\geqslant\bar{p}$. Moreover, since $\bar{p}_k^*=1=\bar{p}_k$,

$$(1+\pi^*)\bar{p}^*M^{*k}=(1+\pi)\bar{p}M^k$$

Then $\pi^*\leqslant\pi$, contradicting part (ⅰ).

For part (ⅲ), it is sufficient to show that $\pi^*\gamma_j^*<\pi\gamma_j$, given that $\pi^*>\pi$ and $q_j=1-\pi\gamma_j$. Let $\bar{p}=p/p_j$ and $\bar{p}^*=p^*/p_j^*$, then $\bar{p}^*>\bar{p}$ according to part (ⅱ)[2]. Moreover,

$$(1+\pi^*)\bar{p}^*(A^{*j}+b^jl_j^*)=\bar{p}_j^*=1=\bar{p}_j=(1+\pi)\bar{p}(A^j+b^jl_j)$$

It follows that $(1+\pi^*)\bar{p}^*bl_j^*=1-(1+\pi^*)\bar{p}^*A^{*j}$ and $(1+\pi)\bar{p}bl_j=1-(1+\pi)\bar{p}A^j$.

By equation (3), $w_jl_j=\frac{\pi}{1+\pi}p_j\gamma_j$, and then

$$\pi\gamma_j=\frac{(1+\pi)w_jl_j}{p_j}=\frac{(1+\pi)pb^jl_j}{p_j}=(1+\pi)\bar{p}b^jl_j$$

Similarly, $\pi^*\gamma_j^*=(1+\pi^*)\bar{p}^*b^jl_j^*$. Therefore,

$$\begin{aligned}\pi\gamma_j-\pi^*\gamma_j^*&=(1+\pi)\bar{p}bl_j-(1+\pi^*)\bar{p}^*bl_j^*\\&=(1+\pi^*)\bar{p}^*A^{*j}-(1+\pi)\bar{p}A^j>0\end{aligned}$$

since $\pi^*>\pi$, $\bar{p}^*\geqslant\bar{p}$, and $A^{*j}\geqslant A^j$. Therefore, $\pi^*\gamma_j^*<\pi\gamma_j$.

For part (ⅳ), since the viable CU-LS technical change is progressive (Roemer, 1977), we have $v^*\ll v$ and hence $v^*B\ll vB$.

Proof of Proposition 2

For part (ⅰ), at the new long-run outcome with $w^*=\alpha w$, we have $p^*=(1+\pi)(p^*A^*+\alpha wL^*)$, i. e., $\frac{1}{(1+\pi)}\frac{p^*}{\alpha}=\frac{p^*}{\alpha}A^*+w\,L^*$, which could be rewritten as

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① For the special case when real wage rate is uniform among sectors, see Bowles (1981) for a simple proof of Okishio's theorem and Dietzenbacher (1989, p. 41) for proof of the converse.

② Remember that $\bar{p}^*>\bar{p}$ means $\bar{p}^*\geqslant\bar{p}$ but $\bar{p}^*\neq\bar{p}$.

$$\frac{1}{(1+\pi)}\left(\frac{p^*}{\alpha},1\right)=\left(\frac{p^*}{\alpha},1\right)\begin{bmatrix}A^* & c^i\\ wL^* & 0\end{bmatrix}$$

where $c^i=\frac{\alpha e_i}{(1+\pi)p_i^*}$ for any $i=1,\cdots n$. That is, the matrix $\begin{bmatrix}A^* & c^i\\ wL^* & 0\end{bmatrix}$ has Frobenius root $\frac{1}{(1+\pi)}$. We show that $pc^i>\frac{1}{(1+\pi)}$. Suppose on the contrary that $pc^i\leqslant\frac{1}{(1+\pi)}$, then

$$\frac{1}{(1+\pi)}(p,1)>(p,1)\begin{bmatrix}A^* & c^i\\ wL^* & 0\end{bmatrix}$$

by the condition of viability, i. e., $pA^*+wL^*<pA+wL=\frac{1}{(1+\pi)}p$. By *Lemma A*, the Frobenius root of the matrix $\begin{bmatrix}A^* & c^i\\ wL^* & 0\end{bmatrix}$ is less than $\frac{1}{(1+\pi)}$. This is a contradiction. Therefore, $pc^i>\frac{1}{(1+\pi)}$ holds, i. e., $\frac{\alpha p_i}{(1+\pi)p_i^*}>\frac{1}{(1+\pi)}$, and hence $\frac{p_i^*}{\alpha}<p_i$ for any $i=1,\cdots n$. That is, $\frac{p^*}{\alpha}\ll p$①.

For (ⅲ), since there is no technical change in any sector other than j and $\pi^*=\pi$, $\gamma_i^*=\frac{\alpha w_il_i}{\pi(p^*A^i+\alpha w_il_i)}$ for any $i\neq j$ by definition. Part (ⅰ) yields $\frac{p^*}{\alpha}<p$, i. e., $p^*<\alpha p$. Therefore,

$$\gamma_i^*=\frac{\alpha w_il_i}{\pi(p^*A^i+\alpha w_il_i)}>\frac{\alpha w_il_i}{\pi(\alpha pA^i+\alpha w_il_i)}=\frac{w_il_i}{\pi(pA^i+w_il_i)}=\gamma_i$$

For (ⅱ), suppose, on the contrary, that $\gamma_j^*\geqslant\gamma_j$. By (ⅲ) we have $\gamma_i^*>\gamma_i$ for all $i\neq j$. Then $\Gamma^*>\Gamma$. Together with $A^*>A$, we have $(1+\pi)A^*+\pi\Gamma^*>(1+\pi)A+\pi\Gamma$. Since both matrices are indecomposable, the Frobenius root of the former must be greater than that of the latter. However, based on Equation (3), both matrices $(1+\pi)A^*+\pi\Gamma^*$ and $(1+\pi)A+\pi\Gamma$ have the same Frobenius root 1. This is a contradiction. Therefore, $\gamma_j^*<\gamma_j$, and then, by the fixed profit rate, $q_j^*=1-\pi\gamma_j^*>1-\pi\gamma_j=q_j$.

Proof of Proposition 3

Let matrix $M(\pi)=(1+\pi)A^*+\pi\Gamma$ and $r(\pi)$ be the Frobenius root. Then $M(\pi)$ is indecomposable and increasing in π. Thus, $r(\pi)$ is continuous and increasing in π. By (3), we have $p^*=p^*[(1+\pi^*)A^*+\pi^*\Gamma]$, which implies that $r(\pi^*)=1$. Moreover,

$$p=p[(1+\pi)A+\pi\Gamma]<p[(1+\pi)A^*+\pi\Gamma]$$

That is, $p<pM(\pi)$, which implies $r(\pi)>1$ by *Lemma A*. Therefore, we have

① For the special case when real wage rate is uniform among sectors, see Fujimoto et al. (1983, p. 125) for a proof based on the so-called Le Chatelier-Samuelson Principle (Fujimoto, 1980), or Dietzenbacher (1988, p. 388) for a direct proof with wages paid post factum.

$r(\pi)>1=r(\pi^*)$ and, thus, $\pi^*<\pi$. Because the wage/profit ratio is unaffected, $q_j^*=1-\pi^*\gamma_j>1-\pi\gamma_j=q_j$.

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